#7 Functions

- In this section we will go over **Functions** by means of logical definitions and mathematical proofs.

Functions

<u>Well-Defined Property:</u> $f: A \longrightarrow B$ is well-defined iff

$$\forall x_1, x_2 \in A, \qquad x_1 = x_2 \Longrightarrow f(x_1) = f(x_2)$$

Example: (not well-defined)

$$f: \mathbb{Q} \longrightarrow \mathbb{Z}\ ,\ \ f\left(\frac{m}{n}\right) = m$$

Negation..

Functions

Injectivity (One-to-one or 1-1): A well-defined function $f: A \longrightarrow B$ is one-to-one iff

$$\forall x_1, x_2 \in A, \qquad f(x_1) = f(x_2) \implies x_1 = x_2$$

Negation..

Functions

Surjectivity (onto): A well-defined function $f: A \longrightarrow B$ is onto iff

$$\forall y \in B, \exists x \in A \; such \; that \; f(x) = y$$

Negation..

Both 1-1 and onto : «Bijective»

Functions

Example

$f: \mathbb{R} \longrightarrow \mathbb{R}$ defined as

$$f(x) = 4x - 1$$

Is f 1-1? Is f onto? Prove.

Example

$f: \mathbb{R} \longrightarrow \mathbb{R}$ defined as

$$f(x) = x^2$$

Is f 1-1? Is f onto? Prove.

Functions

Example

$f: \mathbb{R} - \{1\} \longrightarrow \mathbb{R} - \{1\}$ defined as

$$f(x) = \frac{x+1}{x-1}$$

Is f 1-1? Is f onto? Prove.

.

Functions

Example

$f: \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R} \times \mathbb{R}$ defined as :

$$(x, y) \longmapsto (x + y, x - y)$$

Is f 1-1? Is f onto? Prove.

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