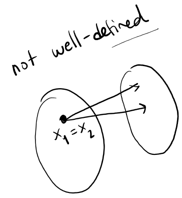
10th Week Friday

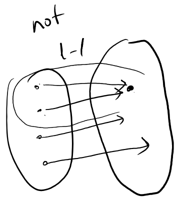
8 Aralık 2023 Cuma 09:33

$f: \underbrace{A}_{\text{domain}} \to \underbrace{B}_{\text{range}}$



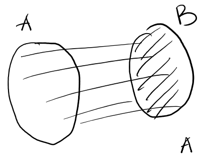
Well-defined : $\forall x_1, x_2 \in A \quad x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$

neg : $\exists x_1, x_2 \in A \quad x_1 = x_2 \wedge f(x_1) \neq f(x_2)$



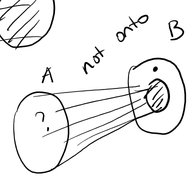
injective **One-to-one (1-1)** : $\forall x_1, x_2 \in A \quad f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

neg : $\exists x_1, x_2 \in A \quad f(x_1) = f(x_2) \wedge x_1 \neq x_2$



surjective **Onto** : $\forall y \in B, \ \exists x \in A : f(x) = y$

neg: $\exists y \in B : \forall x \in A, \ f(x) \neq y$



ex $f: \mathbb{R} \to \mathbb{R} \qquad f(x) = \dfrac{x}{x^2+1}$ Is f 1-1 ? Is f onto ?

$x_1, x_2 \in \mathbb{R}$

Trying the proof: Assume that $f(x_1) = f(x_2)$. (Try to show $x_1 = x_2$)

$$\Rightarrow \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1} \Rightarrow x_1 x_2^2 + x_1 = x_2 x_1^2 + x_2$$

$$\Rightarrow x_1 - x_2 = x_2 x_1^2 - x_1 x_2^2$$

step $\Rightarrow (x_1 - x_2) = x_1 x_2 (x_1 - x_2)$! ? ? ? ? ?

$\Rightarrow x_1 = x_2$ → not a direct result here. $x_1 x_2 = 1$ What if $x_1 - x_2 \neq 0$ $x_1 \neq x_2$

counter example: $f(x_1) = f(x_2) \wedge x_1 \neq x_2$

$x_1 = 3 \neq x_2 = \frac{1}{3}$

$f(x_1) = f(3) = \dfrac{3}{3^2+1} = \dfrac{3}{10}$ $\qquad f(x_2) = f\left(\frac{1}{3}\right) = \dfrac{1/3}{(1/3)^2+1} = \dfrac{1/3}{\frac{1}{9}+1} = \dfrac{1}{3} \cdot \dfrac{9}{10} = \dfrac{3}{10}$

$\Rightarrow f$ is NOT 1-1.

Is f onto? ✗ $\quad f(x) = \dfrac{x}{x^2+1} \quad f: \mathbb{R} \to \mathbb{R}$

Trying to show $\forall y \in \mathbb{R}$ (range), $\exists x \in \mathbb{R}$ (domain) $f(x) = y$

$\forall y \in \mathbb{R}$

$f(x) = y \Rightarrow \dfrac{x}{x^2+1} = y$

leave x alone $x = (\ldots) \in \mathbb{R}$ → domain

$x = x^2 y + y \Rightarrow x^2 y + y - x = 0$

$\Rightarrow x^2 y - 1x + y = 0 \qquad x = \ldots \in \mathbb{R}$

$\Delta = (-1)^2 - 4 \cdot y \cdot y = 1 - 4y^2 \geq 0$? < 0 ✗

$ax^2 + bx + c = 0$ → quadratic eqn.

$x = ?$ $\quad x_{1,2} = \dfrac{-b \mp \sqrt{\Delta}}{2a} \quad b^2 - 4ac \in \mathbb{R}$ ✓

We believe that it is NOT onto.

counter example: $\exists y \in \mathbb{R} : \forall x \in \mathbb{R} \ f(x) \neq y$

counter example

$\exists y \in \mathbb{R}$: $\forall x \in \mathbb{R}$ $f(x) \neq y$

$x = ?$ $x_{1,2} = \frac{-b \mp \sqrt{\Delta}}{2a}$ $b^2 - 4ac$ ✓ $\in \mathbb{R}$

$\Delta = (-1)^2 - 4 \cdot y \cdot y = 1 - 4y^2 \geq 0$? < 0

$1 - 4 \cdot 1^2 = -3 < 0$ $y \in \mathbb{R}$

$y = 1 \in \mathbb{R}$ $\frac{x}{x^2+1} = 1$ Can you find such $x \in \mathbb{R}$?

$x = x^2 + 1 \Rightarrow x^2 - x + 1 = 0$

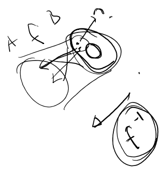
$\Delta = b^2 - 4ac = (-1)^2 - 4 \cdot 1 \cdot 1 = 1 - 4 = -3 < 0 \Rightarrow$ No real x !

$\forall x \in \mathbb{R}$, $f(x) \neq 1$.

Bijective Function : f is 1-1 $\wedge$ f is onto.

✓ ✓

$\Rightarrow$ Inverse of a Function : $f: A \to B$ a well-defined function. $f(x) = y$



When is $f^{-1}: B \to A$ a well-defined function? $f^{-1}(y) = x$

! f should be 1-1 and onto. in order for f^{-1} to be a well-defined function.

15. $f(x) = \dfrac{x+1}{x}$, for all real numbers $x \neq 0$

16. $f(x) = \dfrac{x}{x^2+1}$, for all real numbers x

17. $f(x) = \dfrac{3x-1}{x}$, for all real numbers $x \neq 0$

18. $f(x) = \dfrac{x+1}{x-1}$, for all real numbers $x \neq 1$

$\rightarrow$ Work out yourself to show 1-1? onto?