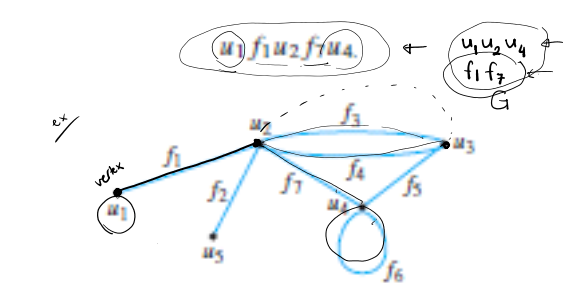


$u_1 f_1 u_2 f_7 u_4$ ← $u_1 u_2 u_4$, $f_1 f_7$

* Flying is NOT allowed.

Travelling from u_1 to u_4 — walk

→ $u_1 f_1 u_2 f_3 u_3 f_4 u_2 f_3 u_3 f_5 u_4 f_6 u_4 f_7 u_2 f_3 u_3 f_5 u_4$.

• Definition

Let G be a graph, and let v and w be vertices in G.

A **walk from v to w** is a finite alternating sequence of adjacent vertices and edges of G. Thus a walk has the form

$$v_0 e_1 v_1 e_2 \cdots v_{n-1} e_n v_n,$$

where the v's represent vertices, the e's represent edges, $v_0 = v$, $v_n = w$, and for all $i = 1, 2, \ldots n$, v_{i-1} and v_i are the endpoints of e_i. The **trivial walk from v to v** consists of the single vertex v.

A **trail from v to w** is a walk from v to w that does not contain a repeated edge.

A **path from v to w** is a trail that does not contain a repeated vertex.

A **closed walk** is a walk that starts and ends at the same vertex.

A **circuit** is a closed walk that contains at least one edge and does not contain a repeated edge.

A **simple circuit** is a circuit that does not have any other repeated vertex except the first and last.

ex: V_1 — trivial walk, closed walk, not a circuit

ex: $V_1 e_1 V_2 e_2 V_3 e_4 V_1$ — circuit; not a trivial walk; a closed walk; s.v = e.v

Trail : repeated edges ✗

path : repeated edges ✗ repeated vertices ✗

circuits : s.v = e.v. ✓ ; at least 1 edge ✓ ; repeated edge ✗

simple circuits : repeated vertices ✗ + ✓

	Repeated Edge?	Repeated Vertex?	Starts and Ends at Same Point?	Must Contain at Least One Edge?
Walk	allowed	allowed	allowed	no
Trail	no	allowed	allowed	no
Path	no	no	no (it means repeated vertex)	no
Closed walk	allowed	allowed	yes ✓	no
Circuit	no	allowed	yes	yes
Simple circuit	no	first and last only	yes	yes

remember trivial walks V_1

All are walk →

a. $v_1 e_1 v_2 e_3 v_3 e_4 v_3 e_5 v_4$

b. $e_1 e_3 e_5 e_5 e_6$ — walk

c. $v_2 v_3 v_4 v_5 v_3 v_6 v_2$ — closed walk ✓

d. $v_2 v_3 v_4 v_5 v_6 v_2$

e. $v_1 e_1 v_2 e_1 v_1$

f. v_1

ex: V_1 — trivial walk, closed walk, ...

a. $v_1 e_1 v_2 e_3 v_3 e_4 v_3 e_5 v_4$

d. $v_2 v_3 v_4 v_5 v_6 v_2$ → trail ✓

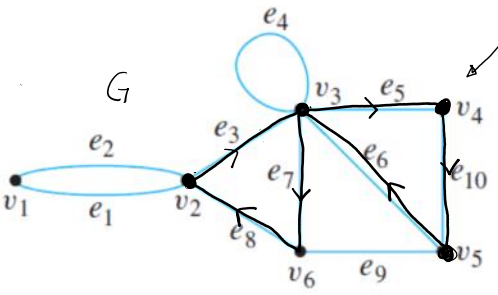
Closed walk ✓
trail ✓
path ✗
→ circuit ✓
→ simple circuit ✓

b. $e_1 e_3 e_5 e_5 e_6$

e. $v_1 e_1 v_2 e_1 v_1$ Closed walk ✓

c. $v_2 v_3 v_4 v_5 v_3 v_6 v_2$ closed walk ✓ trail ✓ circuit ✓

f. v_1 a trivial walk



ex: v_1 trivial walk, closed walk, not a circuit

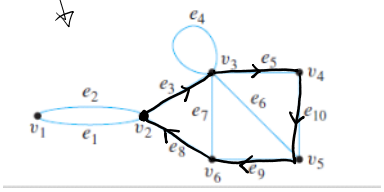
ex: $v_1 e_1 v_2 e_2 v_3 e_4 v_1$ circuit, not a trivial walk, a closed walk, s.v = e.v

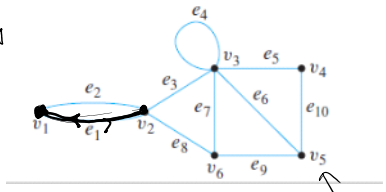
Trail : repeated edges ✗

path: repeated edges ✗ repeated vertices ✗

circuits : s.v = e.v ✓, at least 1 edge ✓, repeated edge ✗

simple circuits: repeated vertices ✗ + ✓





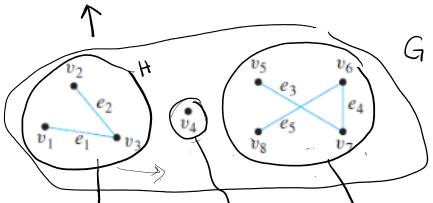
a connected graph.

Connectedness

Two vertices v and w of G are connected if, and only if, there is a walk from v to w. ✓

The graph G is connected if, and only if, any two vertices in G are connected. ✓

not a connected graph



a connected component

another connected component

another connected component

Connected Component

H is a subgraph of G ✓

H is a connected graph ✓

There is no connected subgraph of G which contains H. ✓