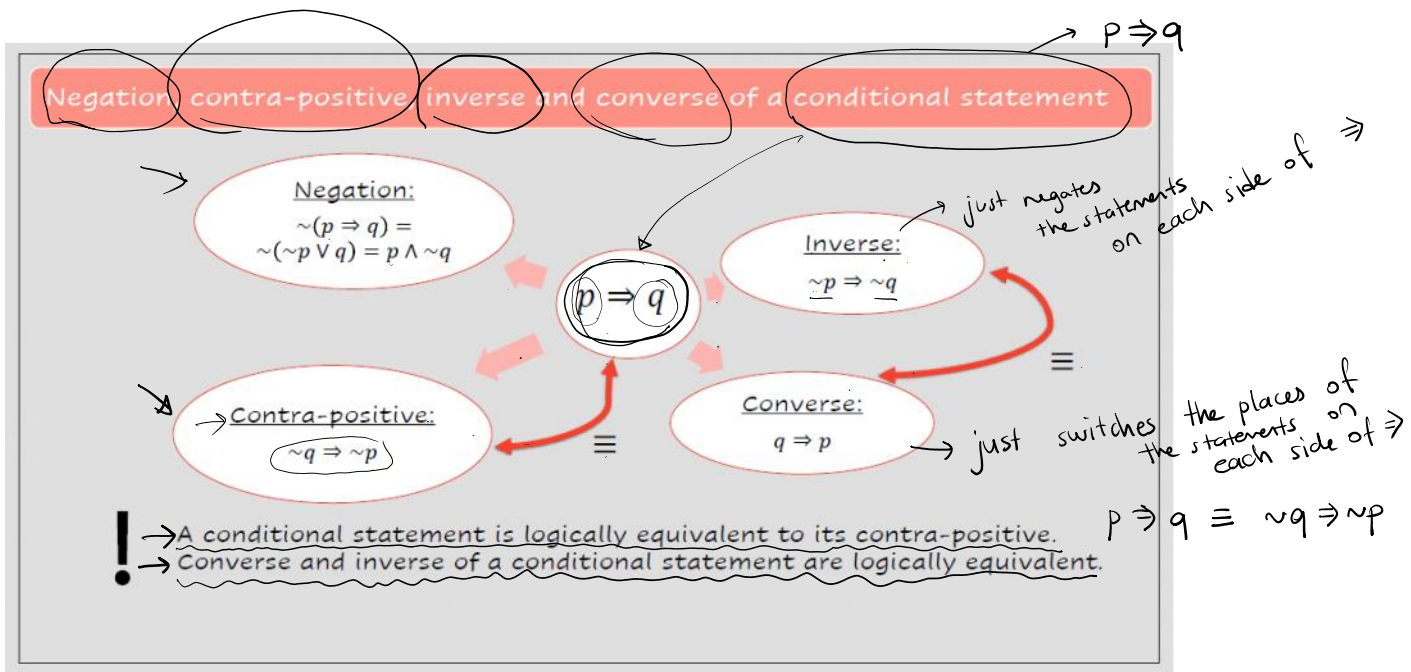


$$p \Rightarrow q$$

<u>p</u>	<u>q</u>	<u>$p \Rightarrow q$</u>
T	T	T
T	F	F
F	T	T
F	F	T

$$(p \vee \sim q) \Rightarrow \sim p$$

<u>p</u>	<u>q</u>	<u>$\sim q$</u>	<u>$\sim p$</u>	<u>$p \vee \sim q$</u>	<u>$(p \vee \sim q) \Rightarrow \sim p$</u>
T	T	F	F	T	F
T	F	T	F	T	F
F	T	F	T	F	T
F	F	T	T	T	T



$$p \Rightarrow q \equiv \sim p \vee q$$

<u>p</u>	<u>q</u>	<u>$p \Rightarrow q$</u>	<u>$\sim p$</u>	<u>$\sim p \vee q$</u>
T	T	T	F	T

we have proved that

p	q	$p \Rightarrow q$	$\sim p$	$\sim p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

we have proved that

$$p \Rightarrow q \equiv \sim p \vee q$$



$$\sim (p \Rightarrow q) = ? \quad \sim (\sim p \vee q) \equiv (\sim(\sim p)) \wedge \sim q$$

$$\equiv p \wedge \sim q$$

$$\sim (p \vee q) = \sim p \wedge \sim q$$

$$\begin{aligned} \sim [(p \wedge q) \Rightarrow (\sim p \vee \sim q)] &\equiv (p \wedge q) \wedge \sim(\sim p \vee \sim q) \\ &\equiv (p \wedge q) \wedge (p \wedge q) \\ &\equiv p \wedge q \end{aligned}$$

Contra-positive of $p \Rightarrow q$: $\sim q \Rightarrow \sim p$

$$p \Rightarrow q \equiv \sim q \Rightarrow \sim p$$

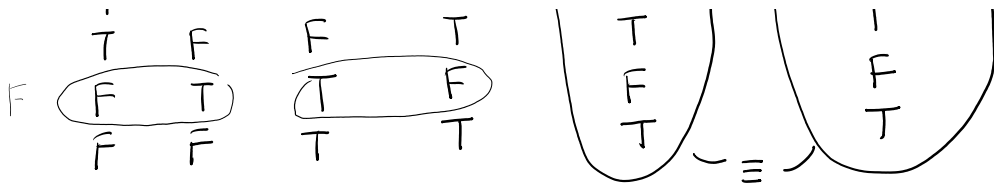
p	q	$\sim q$	$\sim p$	$p \Rightarrow q$	$\sim q \Rightarrow \sim p$
T	T	F	F	T	T
T	F	T	F	F	F
F	T	F	T	T	T
F	F	T	T	T	T

Inverse of $p \Rightarrow q$: $\sim p \Rightarrow \sim q$

Converse of $p \Rightarrow q$: $q \Rightarrow p$

$$\sim p \Rightarrow \sim q \equiv q \Rightarrow p$$

p	q	$\sim p$	$\sim q$	$q \Rightarrow p$	$\sim p \Rightarrow \sim q$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

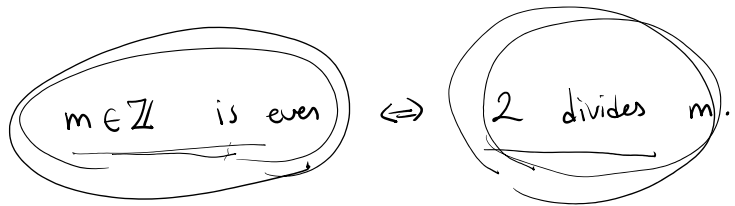


Bi-conditional

$$p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$$

F	T	↓	↓	↓	→
p	q	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Rightarrow q) \wedge (q \Rightarrow p)$	$p \Leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
→ F	F	T	T	T	T

EX/ $p \Leftrightarrow q$



EX

$$p \Rightarrow (\neg q \vee r) \equiv (p \Rightarrow \neg q) \vee r$$

p	q	↓ F	↓	↓	↓	↓
r	$\neg q$	$\neg q \vee r$	$p \Rightarrow \neg q$	$p \Rightarrow (\neg q \vee r)$	$(p \Rightarrow \neg q) \vee r$	$(p \Rightarrow \neg q) \vee r$
T	F	T	F	T	T	T
T	T	F	F	F	F	F
T	F	T	T	T	T	T
T	T	T	T	T	T	T
F	T	T	T	T	T	T
F	F	F	T	T	T	T
F	T	T	T	T	T	T
F	F	T	T	T	T	T

→ If you heat the water it boils. $p \Rightarrow q$
 $\underbrace{\hspace{10em}}_p \quad \underbrace{\hspace{10em}}_q$ $\sim q \Rightarrow \sim p$

→ If the water does not boil then you're not heating it.

If x is a cat x is cute $\checkmark \equiv$
 \equiv if x is not cute then it is not cat. \checkmark