2nd Week

MAT 203E DISCRETE MATHEMATICS

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Valid/Invalid Arguments

Argument:

$$
\begin{aligned}
&p \to q \\
&q \to r \\
\therefore\ &p \to r
\end{aligned}
$$

A sequence of statements:
all statements in an argument except for the final one, are called **premises** (or assumptions or hypotheses). The final statement is called the **conclusion.** The symbol ∴, which is read "therefore," is normally placed just before the conclusion.

Valid / Invalid Arguments

! To say that an argument form is **valid** means that:

if the resulting premises are all true, then the conclusion is also true.

‼ Otherwise the argument form is said to be **invalid**.

Arguments.. checking validity

1. Create the truth table
2. Identify the columns of **premises** and **conclusion**.
3. Specify the «**critical rows**». (Rows where all the premises are True)
4. Look at the truth value of the conclusion at critical rows:
   - If all T : Argument is ***valid.***
   - If there is at least one F: Argument is ***invalid.***

Example

- Determine whether the following argument forms are valid or invalid
- by drawing a truth table, indicating which columns represent the premises and which represent the conclusion,
- and annotating the table with a sentence of explanation.

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow p \\ \therefore p \vee q \end{array} \qquad\qquad \begin{array}{l} p \\ p \rightarrow q \\ \sim q \vee r \\ \therefore r \end{array}$$

Table 2.3.1 Valid Argument Forms

Modus Ponens	$p \to q$ p $\therefore q$	**Elimination**	a. $p \lor q$ $\sim q$ $\therefore p$ b. $p \lor q$ $\sim p$ $\therefore q$
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$	**Transitivity**	$p \to q$ $q \to r$ $\therefore p \to r$
Generalization	a. p $\therefore p \lor q$ b. q $\therefore p \lor q$	**Proof by Division into Cases**	$p \lor q$ $p \to r$ $q \to r$ $\therefore r$
Specialization	a. $p \land q$ $\therefore p$ b. $p \land q$ $\therefore q$		
Conjunction	p q $\therefore p \land q$	**Contradiction Rule**	$\sim p \to c$ $\therefore p$

#3 Predicates and Quantified Statements

- A **predicate** defines property of one or more variables.

P(x) : x is cute.

P: .... Being cute...

Domain(D):

Truth Set: $\{x \in D : P(x)\}$

Example

- Let P be the predicate P(x): «x divides 8» Find the truth sets for the given domains

  a) D=Positive Integers

  b) D=Integers

Universal Quantifier (∀)

- Let Q(x) be a predicate and D the domain of x. A universal statement is a statement of the form "∀x ∈ D, Q(x)."
- It is defined to be true if, and only if, Q(x) is true for every x in D.
- It is defined to be false if, and only if, Q(x) is false for at least one x in D.
- A value for x for which Q(x) is false is called a **counterexample** to the universal statement.

P(x): x is a dog. Q(x): x is cute. D: Zoo

$(\forall x \in D)[P(x) \Rightarrow Q(x)]$

Examples

- $D = \{1,2,3,4,5\}, (\forall x \in D, x^2 \geq x)$. Prove.
- $(\forall x \in \mathbb{R}, x^2 \geq x)$. Disprove.

Existential Quantifier (∃)

- Let Q(x) be a predicate and D the domain of x. An existential statement is a statement of the form "$\exists x \in D$ such that Q(x)."
- It is defined to be true if, and only if, Q(x) is true for at least one x in D.
- It is false if, and only if, Q(x) is false for all x in D.

Examples

- $(\exists x \in \mathbb{Z}^+ : x^2 = x)$. Prove.
- $E = \{5,6,7,8\}, (\exists x \in E : x^2 = x)$. Disprove.

Contra-positive, inverse and converse of a quantified conditional statement

Universal Conditional Statement $(\forall x \in D, P(x) \Rightarrow Q(x))$	
Contra-positive:	$(\forall x \in D, \sim Q(x) \Rightarrow \sim P(x))$
Inverse:	$(\forall x \in D, \sim P(x) \Rightarrow \sim Q(x))$
Converse:	$(\forall x \in D, Q(x) \Rightarrow P(x))$

Negations of Quantified Statements

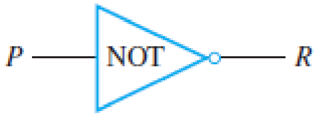
Quantified Statement	Its Negation
Universal Statement $(\forall x \in D, P(x))$	$(\exists x \in D, \sim P(x))$
Universal Conditional Statement $(\forall x \in D, P(x) \Rightarrow Q(x))$	$(\exists x \in D, P(x) \wedge \sim Q(x))$
Existential Statement $(\exists x \in D, P(x))$	$(\forall x \in D, \sim P(x))$
Existential Conditional Statement $(\exists x \in D, P(x) \Rightarrow Q(x))$	$(\forall x \in D, P(x) \wedge \sim Q(x))$

2.4 Application: Digital Logic Circuits

- «Boolean Variable»
- «Boolean Expressions»

Digital Logic Circuits..Gates

NOT

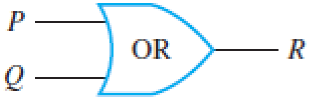


Input	Output
P	*R*
1	0
0	1

AND

Input		Output
P	*Q*	*R*
1	1	1
1	0	0
0	1	0
0	0	0

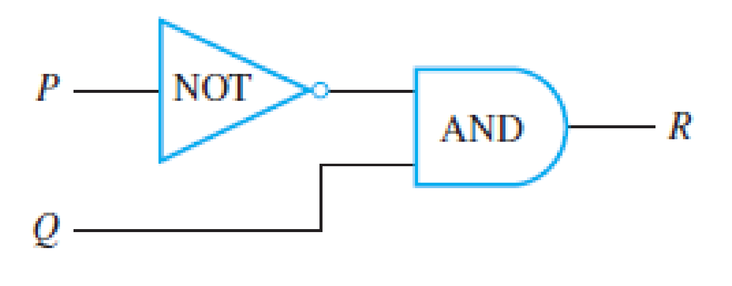
OR



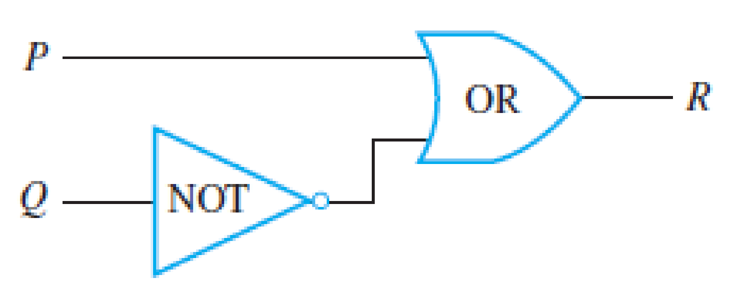
Input		Output
P	*Q*	*R*
1	1	1
1	0	1
0	1	1
0	0	0

The Boolean Expression Corresponding to a Circuit

! Combine from left to right..



Constructing the Input/Output Table for a Circuit



Input		Output
P	*Q*	*R*
1	1	1
1	0	1
0	1	0
0	0	1

The Boolean Expression Corresponding to a Circuit

1. Identify the rows where the Output is 1.
2. For each such row, construct an AND expression that produces a 1 for the exact combination of input values.
3. Use an OR connective at the end.

$$(P \wedge Q \wedge R) \vee (P \wedge {\sim}Q \wedge R) \vee (P \wedge {\sim}Q \wedge {\sim}R)$$

Input			Output
P	*Q*	*R*	*S*
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	0

Finding a Circuit That Corresponds to a Given Boolean Expression

! From outer to the inner; from right to left

$$(P \wedge Q \wedge R) \vee (P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R)$$