4th Week

MAT 203E DISCRETE MATH

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#4. Mathematical Proof

- Mathematical Proof Tecniques
  - Direct Proof
    - Direct Disproof with Counter Example
    - Proof by Cases
  - Indirect Arguments
    - Proof by Contra-positive
    - Proof by Contradiction

Direct Proofs (Proof by Cases)

Theorem 4.4.1 The Quotient-Remainder Theorem

Given any integer n and positive integer d, there exist unique integers q and r such that

$$n = dq + r \quad \text{and} \quad 0 \le r < d.$$

Proof by Cases

Example

- Show that «For any two consequtive numbers, if one of them is odd, the other one is even.»

$$n \text{ is even} \Leftrightarrow \exists k \in \mathbb{Z}: n = 2k$$
$$n \text{ is odd} \Leftrightarrow \exists k \in \mathbb{Z}: n = 2k + 1$$

Example

- Every integer n can be written in one of the forms: n=4q, n=4q+1, n=4q+2 veya n=4q+3, where q is an integer.

Quotient-Remainder Thm

- Given any integer n and a positive integer d, there exist a unique integer couple (q,r) such that $n = dq + r$ and $0 \leq r < d$.

Example

- For any odd number n, n^2 can be written in the form 8m+1, where m is an integer.

Proof by Contra-Positive

Example

- Prove that $(\forall n \in \mathbb{Z}, n^2 \; even \Rightarrow n \; even)$

$(\forall x \in D, P(x) \Rightarrow Q(x))$
$(\forall x \in D, \sim Q(x) \Rightarrow \sim P(x))$

$n \; even \Leftrightarrow \exists k \in \mathbb{Z}: n = 2k$

$n \; odd \Leftrightarrow \exists k \in \mathbb{Z}: n = 2k + 1$

Example

- Prove that for all $a, b, c \in \mathbb{Z}$, if $a \nmid bc \Rightarrow a \nmid b$.

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Proof by Contradiction

1. Assume the negation of the statement.
2. Try to reach to a contradiction.
3. The negation led to a contradiction means the original statement is true.

When to use contradiction?

- To show that there is no element in the domain that satisfies a given condition/property.
- To show that an element does not satisfy a given condition/property.

Example

- Show that the sum of a rational number and an irrational number is an irrational number.

Example

- Show that there is no integer which is both even an odd.