5th Week

MAT 203E DISCRETE MATH

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#5.2 Proof by Mathematical Induction

! In this section, Mathematical Induction, as a proof technique, will be covered on some examples of number theoretical concepts and sequences.

#5.1 Sequences (recap)

Infinite Sequences of Real Numbers

- **General Term**: Given as a_k , stated with an initial value for k.
- a_k : Gives a formula for how the k^{th} term of the sequence is obtained from the value of k.
- The range of values of terms of an infinite sequence may be a finite set.
- A sequence may have different expressions for its general term.

Sums and Products

$$\sum_{k=m}^{n} a_k = a_m + a_{m+1} + a_{m+2} + \cdots + a_n.$$

$$\prod_{k=m}^{n} a_k = a_m \cdot a_{m+1} \cdot a_{m+2} \cdots a_n.$$

Factorials

$$n! = n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1.$$

$$0! = 1$$

$0 \leq r \leq n,$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

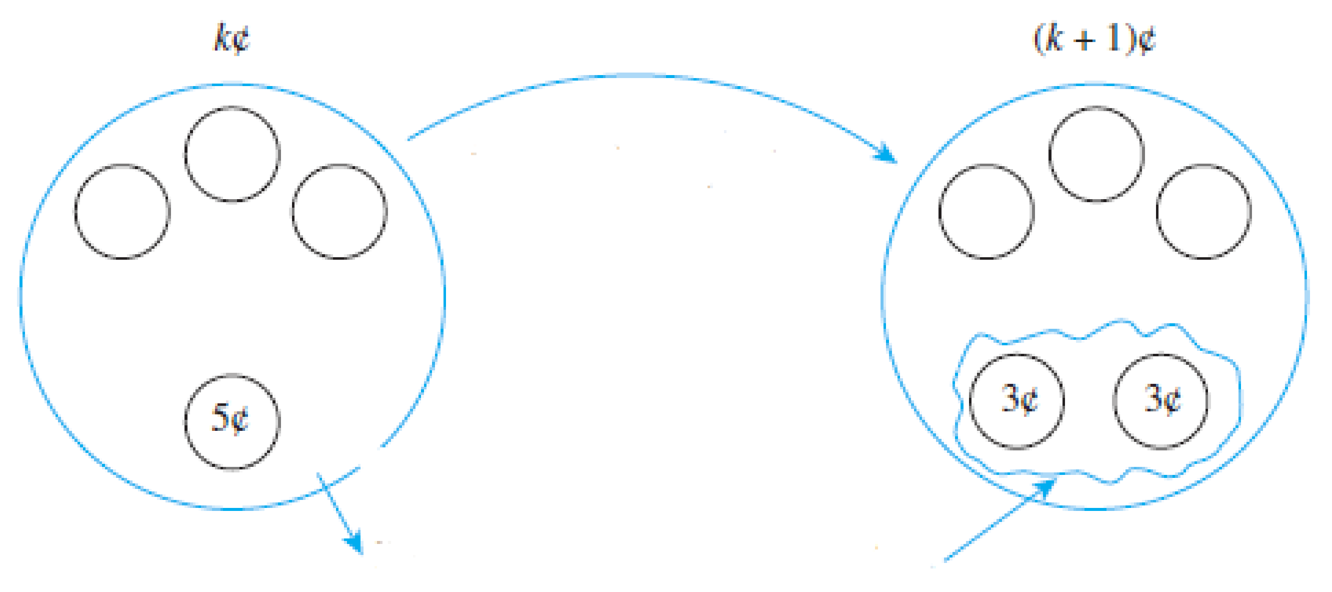
Principal of Mathematical Induction

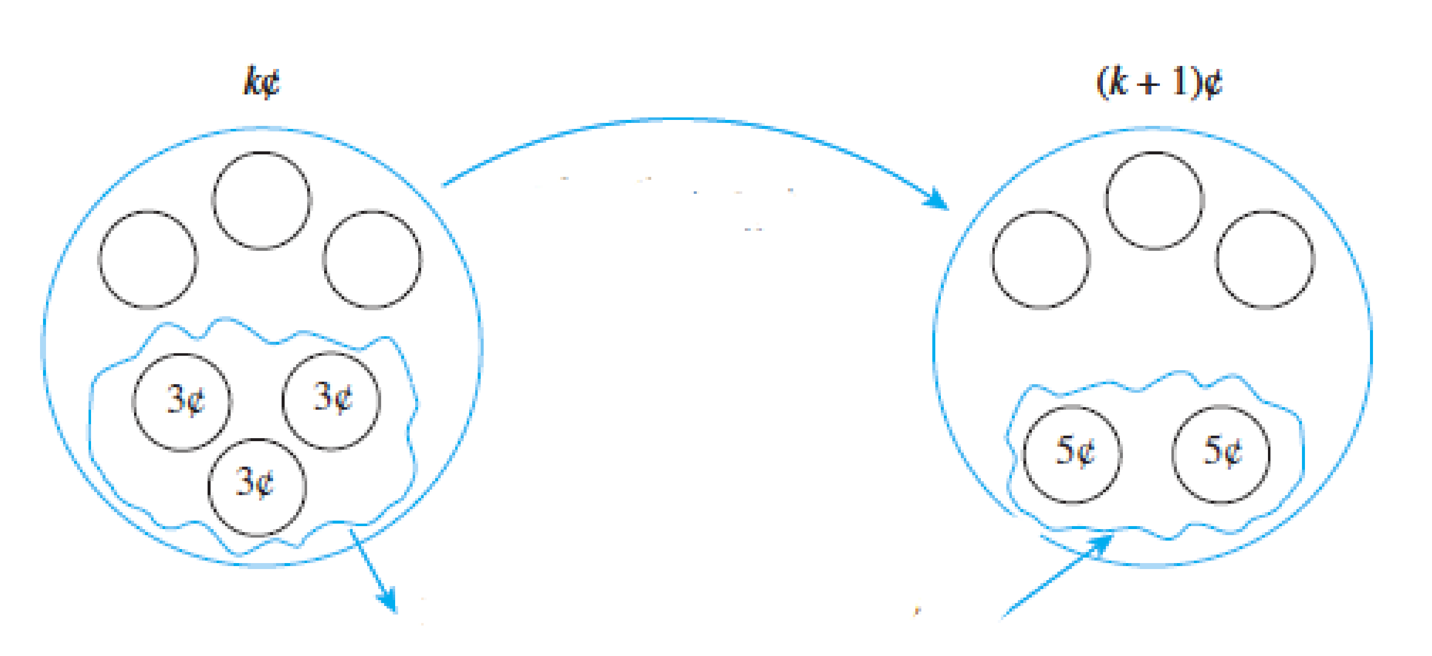
Example (n coin problem)

- $\forall n \geq 8$, n cents can be obtained using 3c and 5c coins.

8¢	3¢ + 5¢
9¢	3¢ + 3¢ + 3¢
10¢	5¢ + 5¢
11¢	3¢ + 3¢ + 5¢
12¢	3¢ + 3¢ + 3¢ + 3¢
13¢	3¢ + 5¢ + 5¢
14¢	3¢ + 3¢ + 3¢ + 5¢
15¢	5¢ + 5¢ + 5¢
16¢	3¢ + 3¢ + 5¢ + 5¢
17¢	3¢ + 3¢ + 3¢ + 3¢ + 5¢

Principal of Mathematical Induction





Proof by Mathematical Induction

Method of Proof by Mathematical Induction

Consider a statement of the form, "For all integers $n \geq a$, a property $P(n)$ is true." To prove such a statement, perform the following two steps:

Step 1 (basis step): Show that $P(a)$ is true.

Step 2 (inductive step): Show that for all integers $k \geq a$, if $P(k)$ is true then $P(k+1)$ is true. To perform this step,

suppose that $P(k)$ is true, where k is any particular but arbitrarily chosen integer with $k \geq a$.

[This suppposition is called the **inductive hypothesis.**]

Then

show that $P(k+1)$ is true.

Proof by Mathematical Induction

1. Equalities (involving Sums/Product)
2. Inequalities
3. Divisibility
4. Sequences (General Terms, recursive sequences)

Example

- Prove for all integers n≥1,

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

Example

- Prove for all integer n≥0,

$$\sum_{i=0}^{n} r^i = \frac{r^{n+1} - 1}{r - 1}$$

Example

- Prove for all integers $n \geq 3$

$$2n + 1 < 2^n$$

Example

- Prove for all integers $n \geq 5$,

$$n^2 < 2^n$$

Example

- Prove for all integers $n \geq 2$,

$$2^n < (n+1)!$$