7th Week

17 Kasım 2023 Cuma 09:24

$A = \{x \in E : P(x)\}$ $B = \{x \in E : Q(x)\}$

Set Theory

$A \subseteq B$: $\forall x, \ x \in A \Rightarrow x \in B$

$A \nsubseteq B$: $\exists x : x \in A \land x \notin B$

$x \in B \Rightarrow Q(x)$

not operations they are statement

$=, \subseteq, \notin$

$\ldots = \emptyset \Rightarrow$ think about using contradiction.

$x \in \Rightarrow$

operation
$$\begin{cases} A \cap B = \{x \in E : x \in A \land x \in B\} \\ A \cup B = \{x \in E : x \in A \lor x \in B\} \\ A - B = \{x \in E : x \in A \land x \notin B\} \\ A^c = \{x \in E : x \notin A\} \end{cases}$$

Union: $\bigcup_{i=1}^{n} A_i = \{x \in E \mid x \in A_i, \exists i \in \{1, \ldots, n\}\}$

Intersection: $\bigcap_{i=1}^{n} A_i = \{x \in E \mid x \in A_i, \forall i \in \{1, \ldots, n\}\}$

A, B, C

$\rightarrow A \cup B \cup C = \{x \in E : x \in A \lor x \in B \lor x \in C\}$

$A_1, A_2, \ldots, A_n$

$\bigcup_{i=1}^{n} A_i = \{x \in E : x \in A_i : \exists i \in \{1, \ldots, n\}\}$

$A \cap B \cap C = \{x \in E : x \in A \land x \in B \land x \in C\}$

$A_1, A_2, \ldots, A_n$ $\bigcap_{i=1}^{n} A_i = \{x \in E : x \in A_i, \forall i \in \{1, \ldots, n\}\}$

Example

- $A_i = \{x \in R \mid -\frac{1}{i} < x < \frac{1}{i}\}$ defined.
- $\bigcup_{i=1}^{3} A_i = ?$ $\bigcap_{i=1}^{3} A_i = ?$

$A_1 = \{x \in \mathbb{R} : -\frac{1}{1} < x < \frac{1}{1}\}$

-1 0 1

$A_2 = \{x \in \mathbb{R} : -\frac{1}{2} < x < \frac{1}{2}\}$

0 $-1/2$ $1/2$

$A_3 = \{x \in \mathbb{R} : -1/3 < x < 1/3\}$

$-1/3$ $1/3$

$A_1 \cup A_2 \cup A_3 = A_1$ $A_1 \cap A_2 \cap A_3 = A_3$

A and B are called "disjoint" sets iff $A \cap B = \emptyset$

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Mutually disjoint

A, B, C are mutually disjoint
$A \cap B = \emptyset$
$A \cap C = \emptyset$
$B \cap C = \emptyset$

$A_1, A_2, \ldots, A_n$ are mutually disjoint $A_i \cap A_j = \emptyset$ $i \neq j$, $\forall i, j \in \{1, \ldots, n\}$

$A_1, A_2, \ldots, A_n$ is a "partition" of B $\Leftrightarrow$ $A_1, A_2, \ldots, A_n$ are mutually disjoint $\land$ $A_1 \cup A_2 \cup \ldots \cup A_n = B$

ex/ divisibility by 3 using Quo-Rem Thm.

$\forall n \in \mathbb{Z}$ $n = 3k$ $n = 3l+1$ $n = 3k+2$, uniquely one of these forms.

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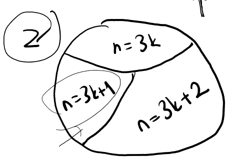
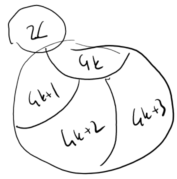


by 4 4 →



Power Set: $\wp(A)$ = Set of all subsets of A

$|\wp(A)| = 2^n$ $n = |A|$

$|A|$ → number of elements in A.

$A = \{1, 2, 3\}$ $|A| = 3$ $|\wp(A)| = 2^3$

$\wp(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$

Ex/ $(A \cap B)^c = A^c \cup B^c$ Prove it.

Proof:

$(A \cap B)^c \subseteq A^c \cup B^c$:

Let $x \in (A \cap B)^c$

$\Rightarrow x \notin A \cap B$ $\sim (x \in A \wedge x \in B)$

$\Rightarrow x \notin A \vee x \notin B$

$\Rightarrow x \in A^c \vee x \in B^c \Rightarrow x \in A^c \cup B^c$

$A^c \cup B^c \subseteq (A \cap B)^c$:

Let $x \in A^c \cup B^c$.

$\Rightarrow x \in A^c \vee x \in B^c$

case 1: $x \in A^c \Rightarrow x \notin A \Rightarrow x \notin A \cap B$

case 2: $x \in B^c \Rightarrow x \notin B \Rightarrow x \notin A \cap B$

$\Rightarrow x \in (A \cap B)^c$

$x \notin A \Rightarrow x \notin A \cap B$

$x \in A \wedge x \in B$

$\therefore (A \cap B)^c = A^c \cup B^c$ ■

Ex/ $(A \cup B)^c = A^c \cap B^c$ Prove it.

$(\subseteq)$: Let $x \in (A \cup B)^c$.

$\Rightarrow x \notin A \cup B$ $\sim (x \in A \vee x \in B)$

$\Rightarrow x \notin A \wedge x \notin B$

$\Rightarrow x \in A^c \wedge x \in B^c$

$\Rightarrow x \in A^c \cap B^c$

$(\supseteq)$: Let $x \in A^c \cap B^c$.

$\Rightarrow x \in A^c \wedge x \in B^c$

$\Rightarrow x \notin A \wedge x \notin B$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \in (A \cup B)^c$. ■

$A - B = A \cap B^c$ → Try to prove it yourself.